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OPTIMAL MODEL OF THE CONTACT FORCE FOR THE COLLISION BETWEEN TWO SOLID BODIES BY ICACO

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ABSTRACT

A collision between bodies is an important phenomenon in many engineering practical applications. The most important problem with the collision analysis is determining the hysteresis damping factor or the hysteresis damping ratio. The hysteresis damping ratio is related to the coefficient of restitution in the collision between two solid bodies. In this paper, at first, the relation between the deformation and its velocity of the contact process is presented. Due to the complexity of the problem under study, a new powerful hybrid metaheuristic method is used to achieve the optimal model. For this purpose, by using imperialist competitive ant colony optimization algorithm, for minimizing the root mean square of the hysteresis damping ratio, the optimal model is determined. The optimal model is entirely acceptable for the wide range of the coefficient of restitution. So, it can be used in hard and soft impact problems.

Kewords: Optimal model; Hybrid metaheuristic method; Collision; Contact force model; Hysteresis damping ratio.

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1. INTRODUCTION

The field of metaheuristic optimization algorithms has recently been attracted to finding the global optimal solution. Many researchers have presented new approaches or improved the existing methods. For example, colliding bodies optimization (CBO) [1, 2], enhanced colliding bodies optimization (ECBO) [3, 4], vibrating particles system [5], enhanced vibrating particles system [6], time evolutionary optimization (TEO) [7], and enhanced time evolutionary optimization (ETEO) [8, 9] are examples of these algorithms. In addition to the hybrid metaheuristic optimization methods due to their superior advantages, have

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been utilized in engineering optimization designs [10]. Researchers have recently proposed many hybrid methods such as imperialist competitive ant colony optimization [11], hybrid improved dolphin echolocation and ant colony optimization algorithm [12], hybrid genetic modified colliding bodies optimization [13, 14], water cycle mine blast algorithm [15], quantum evolutionary algorithm hybridized with enhanced colliding bodies [16], colliding bodies optimization and Morlet wavelet mutation and quadratic interpolation [17], enhanced colliding bodies optimization and artificial neural network [18], Harris Hawks optimization and imperialist competitive algorithm [19] and two different versions colliding bodies optimization such as ECBO and NECBO [20] and etc.

This paper aims to predict an optimal model of contact force for the collision behavior of two objects using a hybrid metaheuristic optimization method that in the following, explanations are provided in this regard.

A collision between two solid bodies is a usual phenomenon in many engineering applications such as mechanisms [21-22], robotics [23], biomechanics [24], railway dynamics [25], and impact dampers [26]. The state of the mechanical system is changed abruptly in the contact events. The velocities and accelerations of the colliding bodies are discontinuous in these problems. This discontinuity causes the nonlinearity of the dynamic behavior of multibody systems. When two bodies impact each other, the contact force relationship between them must be satisfied. Therefore, the contact force model is an important issue in the contact process.

The most important problem in the nonlinear viscoelastic model is determining the hysteresis damping factor or the hysteresis damping ratio. Many researchers have worked on this matter. Some researches such as Ristow [27], Lee and Herrmann [28], Schäfer et al. [29], Bordbar and Hyppänen [30], as well as Zhang and Sharf [31] determined the hysteresis damping factor by the experimental tests. Herbert and McWhannell [32], Gonthier et al. [33], as well as Zhang and Sharf [34] proposed an exact equation for determining the hysteresis damping factor. This exact equation is a nonlinear function between the hysteresis damping factor and the physical parameters of the contact process such as the coefficient of restitution. This equation doesn’t have an explicit solution but can be solved numerically. Hunt and Crossley [35], Lee and Wang [36], Kuwabara and Kono [37], Lankarani and Nikravesh [38], Tsuji et al. [39], Brilliantov et al. [40], Marhefka and Orin [23], as well as Gharib and Hurmuzlu [41] considered a simple assumption and obtained an explicit expression for the hysteresis damping factor. Some researchers such as Flores et al [42], Hu and Guo [43], and Safaeifar and Farshidinafar [44] considered an expression for the relation between the deformation and its velocity. These researchers obtained an explicit expression between the hysteresis damping factor and the coefficient of restitution.

The literature review on the contact force models of collision between two solid bodies shows that these models were typically proposed by comparing the obtained results and so far, no optimal model has been presented in this regard. For this reason, in this research, a new hybrid meta-heuristic optimization algorithm is applied due to the complexity, nonlinearity, and high computational cost of the problem of the contact

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force model for the collision between two solid bodies.

The imperialist competitive ant colony optimization method (ICACO) is a hybrid strong metaheuristic optimization method that has excellent advantages including comfortable performance, a low number of tuning parameters, a high ability to the analysis of complex engineering problems, and, a fast convergence rate [11].

In this research, an optimal new model for the contact force between two colliding bodies is acquired by using imperialist competitive ant colony optimization. For this purpose, at first, the mathematical modeling of a contact process has been reviewed and then the characteristics and implementation of ICACO are presented. Finally, the optimal model of the contact force for the collision between two solid bodies is provided by using the ICACO method.

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2. MATHEMATICAL MODELING

Collision is a physical process which can be occurred in our daily life. Impact refers to the collision between two solid bodies which it is determined by the generation of large contact forces acting over a short interval of time.

Collisions between bodies are governed by the conversation laws of momentum [1,45]. Fig. 1 shows collisions between two solid bodies [46]. In this figure two bodies with masses of m_1 and m_2 are moving in 1-dimensional space with velocities v_1 and v_2 before impact. For collision conditions these velocities must related so that $v_1 > v_2$. These two bodies collide with each other and their velocities after impact (v_1' and v_2') related so that $v_1' < v_2'$.

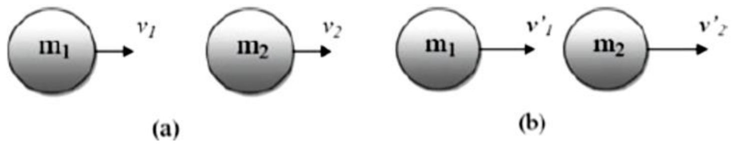

Figure 1. The velocities before and after impact in collision between two solids

In the impact process, two solid bodies act a force F to each other which can be shown in Fig. 2 where the net external force is zero.

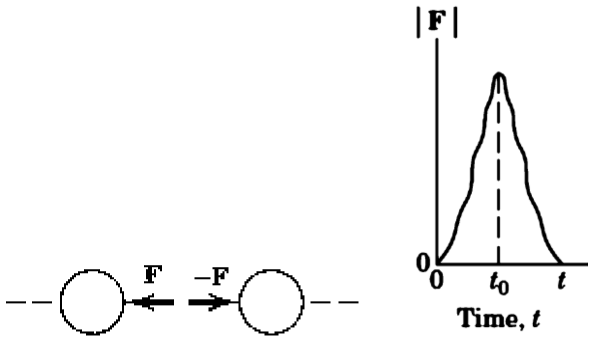

Figure 2. The acting force in collision between two solids

Provided that there are no net external forces acting upon these two colliding solids, the momentum of these solids before the collision equals to the momentum of them after the collision, and can be expressed by the following equation [3,45]:

$$m_1\, v_1 + m_2\, v_2 = m_1\, v_1{}' + m_2\, v_2{}' \tag{1}$$

Likewise, the conservation of the total kinetic energy is expressed as:

$$\frac{1}{2} m_1\, v_1{}^2 + \frac{1}{2} m_2\, v_2{}^2 = \frac{1}{2} m_1\, v_1{}'^2 + \frac{1}{2} m_2\, v_2{}'^2 + Q \tag{2}$$

where Q is the loss of kinetic energy due to the impact [3,46].
The formulas for the velocities after a one-dimensional collision can be determined as follows [20]:

$$\begin{aligned} v_1{}' &= \frac{(m_1 - c_r m_2)v_1 + (m_2 + c_r m_2)v_2}{m_1 + m_2} \\ v_2{}' &= \frac{(m_2 - c_r m_1)v_2 + (m_1 + c_r m_1)v_1}{m_1 + m_2} \end{aligned} \tag{3}$$

where c_r is the Coefficient of Restitution (COR) of the two colliding bodies, defined as the ratio of relative velocity of separation to relative velocity of approach [47]:

$$c_r = \frac{|v_2{}' - v_1{}'|}{|v_1 - v_2|} \tag{4}$$

According to the coefficient of restitution, there are two special cases of any collision as follows [47]:

1) A perfectly elastic collision is defined as the one in which there is no loss of kinetic energy in the collision ($Q = 0$ and $c_r = 1$).

2) An inelastic collision is the one in which part of the kinetic energy is changed to some other form of energy in the collision. Some of kinetic energy will be converted to other forms of energy. In this case, coefficient of restitution does not equal to one ($Q \neq 0$ and $c_r \leq 1$). For the most real objects, the value of c_r is between 0 and 1 [47].

Fig. 3 shows the contact between two solid spheres. In this figure, x_1, x_2, δ_1, and δ_2 represent the displacements of the center of mass and the deformations of both spheres, respectively, just like the parameters m_1, m_2, R_1, and R_2 that represent the masses and the radii of both spheres [44]. In this figure, the total deformation δ is the sum of deformations of both spheres, i.e., $\delta = \delta_1 + \delta_2$.

Many researchers proposed a nonlinear viscoelastic contact force model for the collision between two solid bodies which can be expressed as [44]

$$F = K\delta^n + C\delta^n\dot{\delta} \tag{5}$$

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where K represents the generalized stiffness parameter and δ is the relative normal deformation between the two contacting bodies. For the two contacting spheres, the generalized parameter K is a function of the radii of the spheres and the material properties. Under this condition, the generalized parameter K can be expressed as [27,39,48]

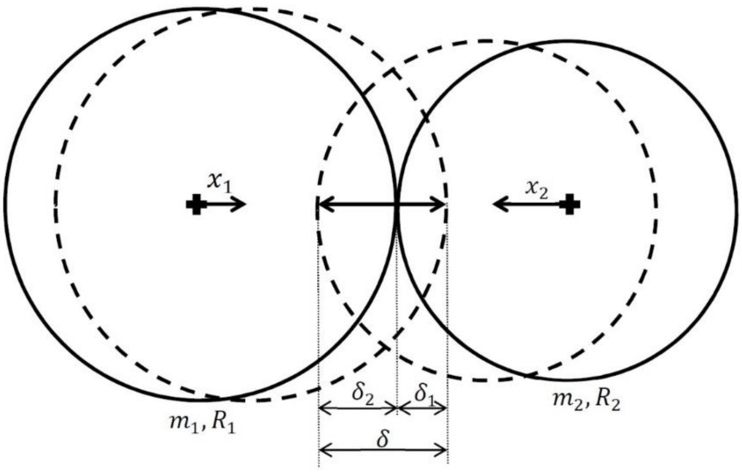

Figure 3. The contact between two spheres

$$K = \frac{4}{3\pi(\sigma_1 + \sigma_2)}\sqrt{\frac{R_1 R_2}{R_1 + R_2}} \tag{6}$$

where R_1 and R_2 are the radii of the spheres, and the material parameters of the spheres, σ_1 and σ_2 , are given by

$$\sigma_i = \frac{1 - {v_i}^2}{\pi E_i} \qquad i = 1,2 \tag{7}$$

where the variables v_i and E_i are the Poisson's ratios and the Young moduli of the spheres, respectively.

The exponent n in Eq. (5) is usually set to $3/2$ in the Hertz model [38] and C in this equation is the hysteresis damping factor. So, the contact force between the two colliding spheres can be expressed as [44]

$$F = K\delta^{3/2} + C\delta^{3/2}\dot{\delta} \tag{8}$$

The relations between the hysteresis damping factor and the coefficient of restitution in some earlier contact force models are listed in Table 1.

Each contact process consists of two phases. The first phase is named compression, while the other phase is named restitution [48]. The two spheres come in contact and reach their maximum deformation during the compression period. In this period, the deformation velocity is reduced from its initial value to zero. The two spheres separate from each other

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during the restitution period, in which the deformation velocity is increased to its maximum value.

Table 1. The hysteresis damping factor in some earlier contact force models

The contact force model	The hysteresis damping factor (C)
Hunt and Crossley [35]	$\frac{3(1-c_r)}{2}\frac{K}{\dot{\delta}^-}$
Lankarani and Nikravesh [38]	$\frac{3(1-{c_r}^2)}{4}\frac{K}{\dot{\delta}^-}$
Flores et al. [42]	$\frac{8\,(1-c_r)}{5\,c_r}\frac{K}{\dot{\delta}^-}$
Gharib and Hurmuzlu [41]	$\frac{1}{c_r}\frac{K}{\dot{\delta}^-}$
Hu and Guo [43]	$\frac{3\,(1-c_r)}{2\,c_r}\frac{K}{\dot{\delta}^-}$
Safaeifar and Farshidianfar [44]	$\frac{5\,(1-c_r)}{4\,c_r}\frac{K}{\dot{\delta}^-}$

The relation between the hysteresis damping factor and the coefficient of restitution can be expressed as [34, 44]

$$\dot{\delta}^-(1+c_r)+\frac{K}{C}ln\left(\frac{K-Cc_r\dot{\delta}^-}{K+C\dot{\delta}^-}\right)=0 \tag{9}$$

For simplicity, the hysteresis damping ratio is defined as [44]

$$h_r=\frac{C\dot{\delta}^-}{K} \tag{10}$$

So, the relation between the hysteresis damping ratio and the coefficient of restitution can be expressed as [34,44]

$$h_r(1+c_r)=ln\left(\frac{1+h_r}{1-c_rh_r}\right) \tag{11}$$

This relation is the exact equation between the hysteresis damping ratio and the coefficient of restitution.

This equation has no explicit solution but can be solved numerically [44]. To derive an explicit expression for the relation between the hysteresis damping ratio and the coefficient of restitution, a simpler relation between the deformation and its velocity can be considered.

The relation between the hysteresis damping ratio and the coefficient of restitution in the numerical model and the earlier models was shown in Fig. 4.

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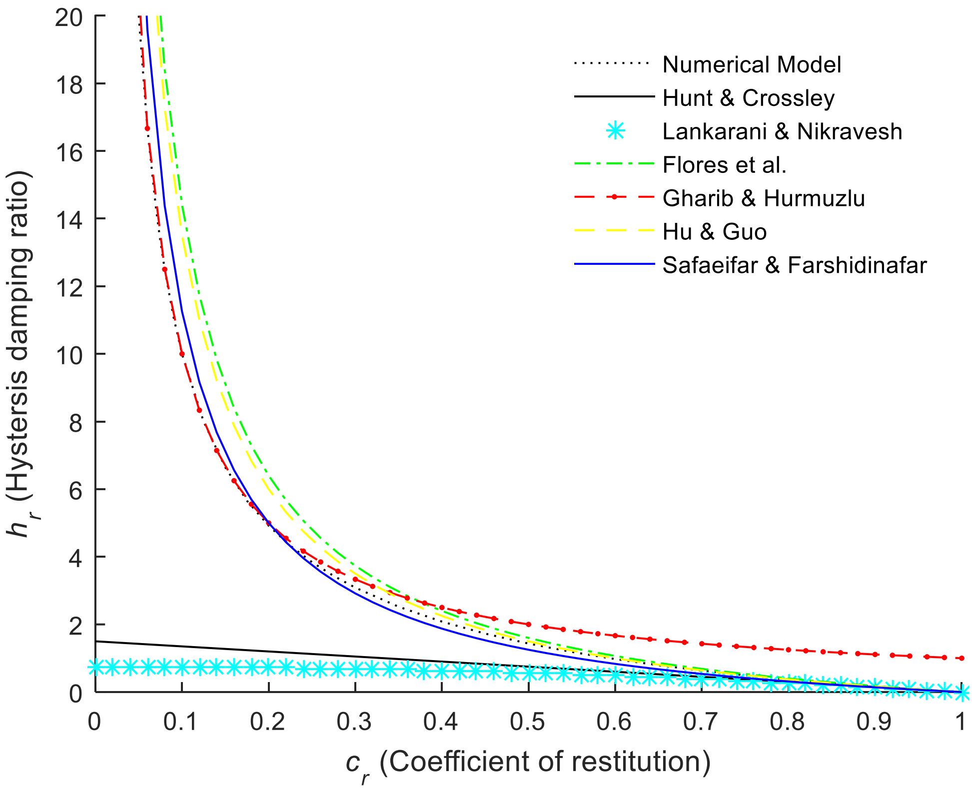

Figure 4. The hysteresis damping ratio versus the coefficient of restitution in the numerical model and the earlier models

As shown in Fig. 4, Safaeifar and Farshidinafar's model [44] and the numerical model almost have similar behavior for most values of the coefficient of restitution but it isn’t optimal and can be improved. When the coefficient of restitution is nearing zero, the hysteresis damping ratio in this model becomes infinite. The hysteresis damping ratio becomes zero when the coefficient of restitution is equal to unity. These two points are logical from the physical view of the contact process.

The numerical model is nearly equivalent to the real contact force model. So, for almost values of coefficient of restitution, Safaeifar and Farshidinfar model is closer to reality.

It can be observed that the behavior of Flores et al. [42], Hu and Guo [43] and Safaeifar and Farshidinafar [44] models are almost similar to the numerical model. Flores et al. used the solution of the Kelvin_Voigt model and considered the relation between the deformation and its velocity as [42].

$$\frac{\dot{\delta}}{\dot{\delta}^{-}} = \left(1 - \left(\frac{\delta}{\delta_{max}}\right)^2\right)^{\frac{1}{2}} \tag{12}$$

Hu and Guo used the solution of the Hertz model and considered the relation between the deformation and its velocity as [43]

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$$\frac{\dot{\delta}}{\dot{\delta}^{-}}=\left(1-\left(\frac{\delta}{\delta_{max}}\right)^{\frac{5}{2}}\right)^{\frac{1}{2}} \tag{13}$$

Safaeifar and Farshidinfar considered a parametric expression for the relation between the deformation and its velocity in the compression period [44]

$$\frac{\dot{\delta}}{\dot{\delta}^{-}}=\left(1-\left(\frac{\delta}{\delta_{max}}\right)^{a}\right)^{\frac{1}{b}} \tag{14}$$

where a and b are two unknown constants. Safaeifar and Farshidinfar determined a and b using minimizing the root mean square (rms) of the percentage error of the hysteresis damping ratio of their model with respect to the numerical model. This minimizing is done by comparing results of some determined values for a and b [44]. Safaeifar and Farshidinfar model can be expressed as [44]

$$\frac{\dot{\delta}}{\dot{\delta}^{-}}=\left(1-\left(\frac{\delta}{\delta_{max}}\right)^{\frac{3}{2}}\right)^{\frac{2}{11}} \tag{15}$$

Introducing $\mathrm{I}=\int_0^1 \mathrm{y}^{3/2}(1-\mathrm{y}^{\mathrm{a}})^{\frac{1}{\mathrm{b}}}\mathrm{dy}$, the hysteresis damping factor can be obtained as [44]

$$C=\frac{2}{5I}\frac{(1-c_r)}{c_r}\frac{K}{\dot{\delta}^{-}} \tag{16}$$

The hysteresis damping ratio in the new model can be expressed as [44]

$$h_r=\frac{2}{5I}\frac{(1-c_r)}{c_r} \tag{17}$$

where I is a function of two unknown constants a and b.

As shown in Eqs. (10), (11), and (14), it is clear that the relation between the deformation and deformation velocity is related to the modulus of elasticity, Poisson's ratio, radius, and mass of two colliding bodies. So, parameters a, b and I are related to the modulus of elasticity, Poisson's ratio, radius, and mass of two colliding bodies.

3. IMPERIALIST COMPETITIVE ANT COLONY OPTIMIZATION (ICACO)

Imperialist competitive ant colony optimization (ICACO) is a hybrid meta heuristic

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optimization method using the socio-political mutation of humans as an origin of inspiration [11]. ICACO consists of a combination of imperialist competitive algorithm (ICA) and ant colony optimization (ACO). The primary version of ICA originates randomly and each individual of them is named ‘country’. The population is then classified into two groups, including imperialists and colonies of imperialists [49-50]. Colonies are under the control of an imperialist. The imperialists had much power (the more optimized countries) with respect to colonies so all the colonies were distributed between imperialists based on their authority. Every empire has involved an imperialist with its colonies. During the optimization procedure, colonies started to move to their imperialist. In an empire, if a colony has better power than that of an imperialist, the position of the colony and its associated imperialist must be exchanged. To estimate the total power of an empire, the percentage of the average power of colonies and the imperialist of it is regarded. The empires that cannot enhance their power in the imperialistic competition will gradually become weaker and will eliminate eventually. Therefore, their colonies would join into other empires and, stronger empires were constructed. The competition between the empires is performed until only one empire remains. Finally, in this empire, the characteristics and power of all the colonies and imperialists will be the same.

The steps of the optimization process by ICA are as follows:

Step 1: Design initial country positions by Eq. (18)

$$X_{k,l}^{(0)} = U_{x_k}^{b} + r.\left(U_{x_k}^{b} - L_{x_k}^{b}\right), \left| \begin{array}{l} k = 1,2, \dots , N_{DV} \\ l = 1,2, \dots , N \end{array} \right. \tag{18}$$

where $X_{k,l}^{(0)}$ is the initial value of the *k*th design variable for the *l*th country; $U_{x_k}^{b}$ and $L_{x_k}^{b}$ are side constraint; r is a random number between zero and unity, N and N_{DV} are the entire number of countries and design variables, respectively.

Step 2: Define imperialist and colonies

After the objective function of the initial countries is estimated, the empires are constructed. So, some of the countries with high power will be chosen as the imperialist states and the rest of them will be the colonies.

Step 3: Move the colonies toward their relevant imperialists

The colony movement towards the imperialist was defined as Eq. (19):

$$\{X\}_{new} = \{X\}_{old} + U(0, \beta \times d) \times \{V_1\} \tag{19}$$

where U has a random cost that is distributed evenly between zero and $\beta \times d$. d and β are the distance between imperialist and colony, and a scaler parameter that is greater than 1 respectively. $\{V_1\}$ is a unit vector between the locations of the colony and the relevant imperialist.

The random parameter θ is regarded the direction of movement for growing the searching space around the imperialist.

$$\theta = U(-\gamma, +\gamma) \tag{20}$$

where γ is a parameter that modifies the change from the main direction.

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Step 4: Change the location of the best colony and its imperialist

In this step, if a colony is designed with more power than the associated imperialist, the location of the imperialist and colony will be exchanged.

Step 5: Estimate the total power of an empire

The total power of an empire is estimated based on both the power of the imperialist and its colonies as Eq. (21).

$$TC_l = f^{(imp,l)} + \xi . \frac{\sum_{i=1}^{NC_l} f^{(col,l)}}{NC_l} \tag{21}$$

where TC_l is the total power of the lth empire, f is the fitness function, NC_l is the number of empires, ξ has a nonnegative value and less than one.

Step 6: Select the weakest colony in the least powerful empire and join with the most powerful empire.

Step 7: Collapse of the empires without colonies

Step 8: Stop the optimization algorithm if the stop criteria are satisfied otherwise go to step two.

The graphical steps imperialist competitive algorithm is shown in Fig. 5 [10]. In this figure, the stars represent the imperialists and the circles depict the colonies.

The most important defect of the imperialist competitive algorithm is not balancing among exploration and exploitation phases in optimization process. In order to fix this weakness, the two methods ICA and ACO were combined, and a new hybrid method called imperialist competitive ant colony optimization (ICACO) was developed [11]. In ICACO, at first, initial ants (N_{Col}) are produced. The locations of these ants are created around their associated imperialist.

$$Ant_{l,n}^{k} = N(imp_N, \sigma), \left| \begin{matrix} l = 1,2, \dots, N.C_n \\ n = 1,2, \dots, N_{imp} \end{matrix} \right. \tag{22}$$

where $N.C_n$ and N_{imp} are the number of colonies of the nth empire and imperialist respectively. The solution $Ant_{l,n}^{k}$ is created by ant lth in empire nth in the iteration k;

$$\begin{gathered} Ant^k = \left[Ant_{1,1} \dots Ant_{N.C_1,1}\, Ant_{1,2} \dots Ant_{N.C_1,2} \dots Ant_{N.C_{imp},N_{imp}}\right]^T, \\ N.C_1 + N.C_2 + \dots + N.C_{N_{imp}} = N_{col} \end{gathered} \tag{23}$$

$N(imp_N, \sigma)$ has a normal random value that is distributed with variance σ and average value imperialist nth. Variance σ is determined by Eq. (24).

$$\sigma = (U^b - L^b) \times \eta \tag{24}$$

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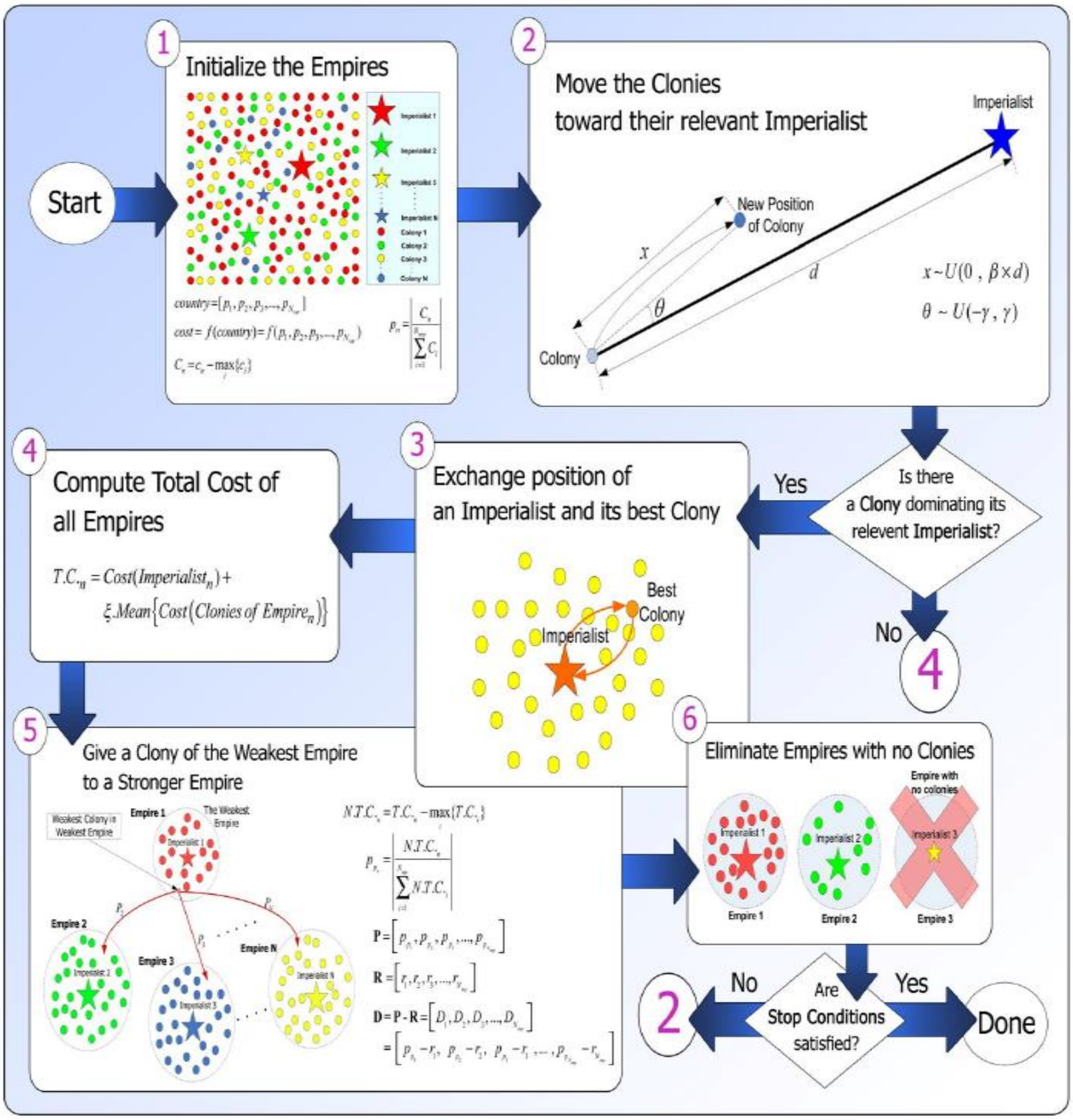

Figure 5. The graphical steps of ICA [10]

where L^b and U^b are the lower and upper bound respectively. η is used to regulate the moving step which initially is equal to one and by approaching the optimum point, decreases gradually and finally tends to zero. After producing Ants, the value of their fitness function of them ($f(Ant_{l,n}^k)$) is determined. If $f(colony_{l,n}^k)$ is in the feasible domain and it is more than $f(Ant_{l,n}^k)$, the location of ant *l*th in empire nth ($Ant_{l,n}^k$) is exchanged with the location $colony_{l,n}^k$ (the current location of colony *l*th in empire *n*th).

The hybridization of ant colony optimization and imperialist competitive algorithm creates a balance between the exploration and exploitation phases. Fig. 4 presents the flowchart of the ICACO algorithm [26]. In this figure, the blue parts are inspired by ACO.

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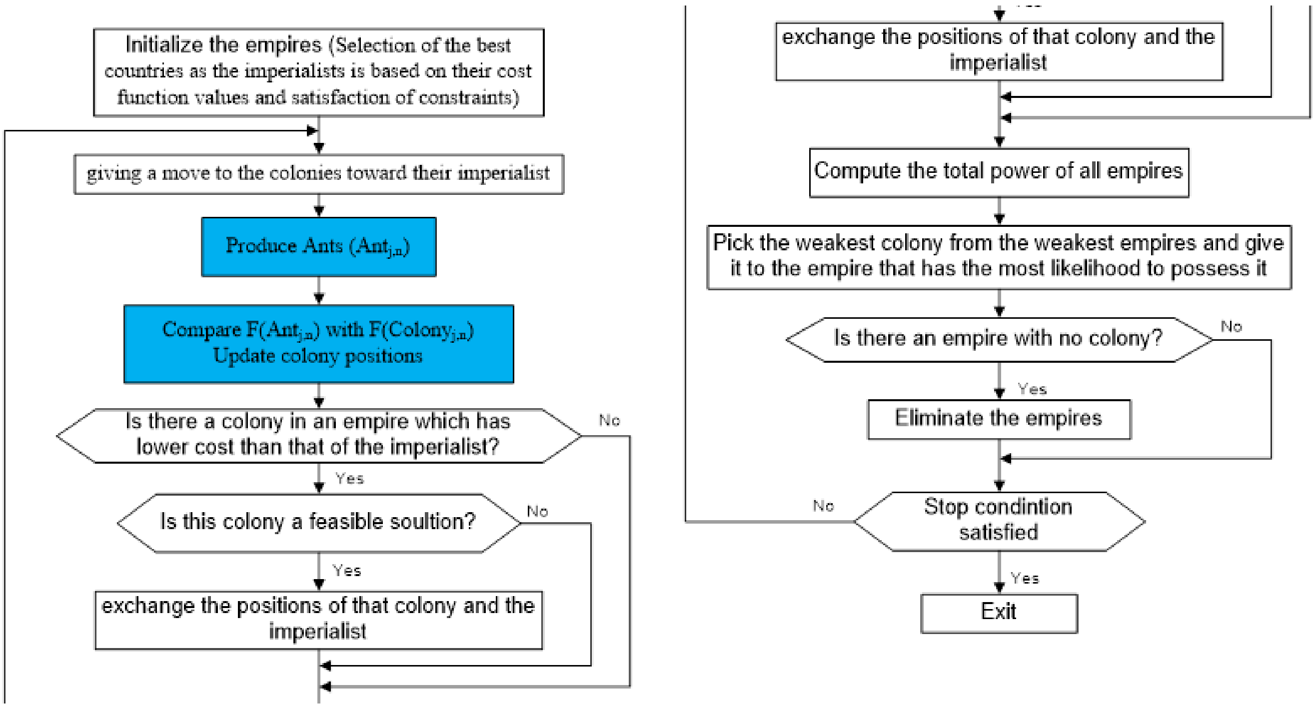

Figure 6. Flowchart of the imperialist competitive ant colony optimization [11]

4. OPTIMAL MODEL OF THE CONTACT FORCE

In this section, the optimal model of the contact force for the collision between two solid bodies will be determined by using ICACO. For this purpose, at first two codes in MATLAB consist of analyzing the main problem, and the optimization process with ICACO are produced. Next, by linking these two codes, the optimal model of the problem is obtained.

In the optimization process, the parameters tuning such as ξ, β, γ are 0.1, 2.0, and 0.3 respectively. The number of initial population and imperialist are 20 and 3 respectively. The positions of the initial population and imperialists are shown in Fig. 7. As mentioned earlier, the stars and circles depict the imperialists and colonies respectively. The more power of an empire, the bigger star is shown for it. The positions of countries in the 12th iteration and final iteration are shown in Fig. 8. As it can be seen, at this iteration, there exists just one empire and all the colonies belong to one imperialist. Then all the colonies move toward the remained imperialist and will have the same cost and power as their imperialist. The convergence criterion of reaching 50 iterations has been considered. The convergence curve of reaching the optimal model by ICA and ICACO is presented in Fig. 9. As can be seen, the ICACO method has a faster convergence rate than ICA.

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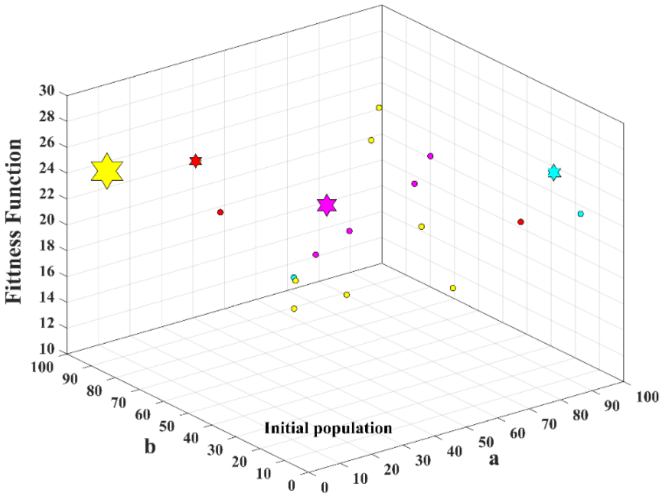

Figure 7. The positions of the initial population by ICACO

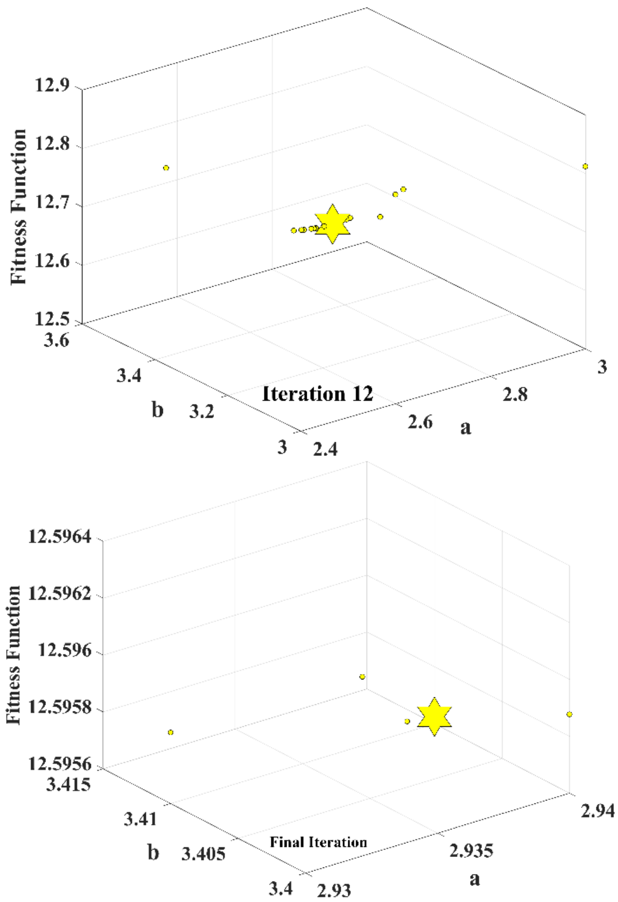

Figure 8. The positions of the population by ICACO in iteration 12 and the final iteration

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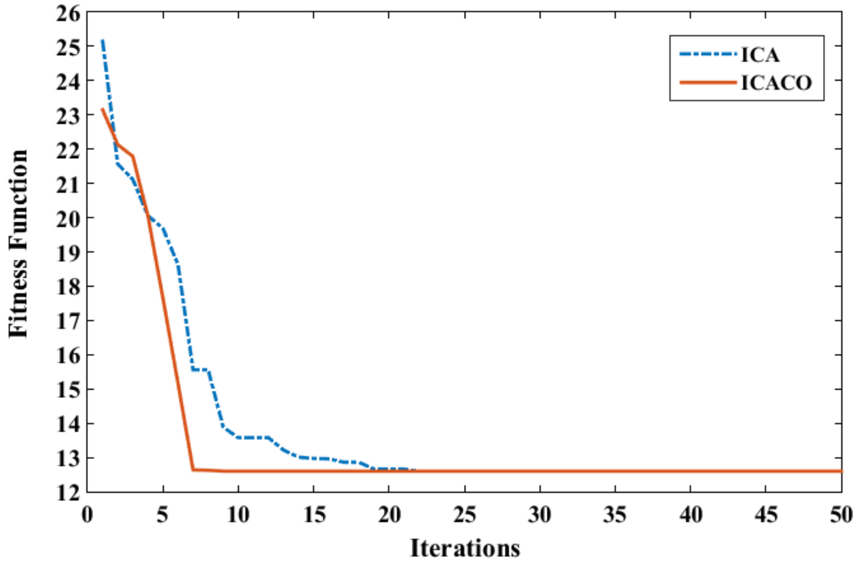

Figure 9. The convergence curve for optimal model by ICA and ICACO

The optimization process shows that the rms of the percentage error is minimized at $a = 2.939$ and $b = 3.409$. This minimized value is 12.595.The statistical results obtained from 50 independent runs of optimization algorithm ICACO are presented in Table 2. The closeness of the values of the best, average, and worst solutions obtained, as well as the very small standard deviation, show the power of the proposed optimization method in reaching the optimal model.

Table 2 The statistical results of optimal models by ICACO at 50 independent runs

Best	Average	Worst	Standard deviation
12.5955	12.5956	12.6789	2.7339e-07

It is noted that the rms of the percentage error of the Safaeifar and Farshidianfar model [44] is 12.596. So, the relation between the deformation and its velocity can be expressed as

$$\frac{\dot{\delta}}{\dot{\delta}^{-}} = \left(1 - \left(\frac{\delta}{\delta_{max}}\right)^{2.939}\right)^{\frac{1}{3.409}} \tag{25}$$

Thus, parameter I can be expressed as

$$I = \int_0^1 \left(y^{2.939}\left(1 - y^{3/2}\right)^{1/3.409}\right) dy \tag{26}$$

which can be evaluated as

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$$I = 0.31788 \tag{27}$$

Combining relations (17) and (27), the hysteresis damping ratio can be obtained as Eq. (28).

$$h_r = 1.25834 \frac{(1 - c_r)}{c_r} \approx \frac{200}{159} \frac{(1 - c_r)}{c_r} \tag{28}$$

The hysteresis damping factor in the proposed model can be obtained as Eq. (29).

$$C = \frac{200}{159} \frac{(1 - c_r)}{c_r} \frac{K}{\dot{\delta}^-} \tag{29}$$

Thus, the proposed model of the contact force can be expressed as Eq. (30).

$$F = K\delta^{3/2} \left(1 + \frac{200}{159} \frac{(1 - c_r)}{c_r} \frac{\dot{\delta}}{\dot{\delta}^-} \right) \tag{30}$$

The relation (30) is called the optimal model. The relation between the hysteresis damping ratio and the coefficient of restitution in the optimal model, the numerical model, as well as Flores et al. [42], Hue and Guo [43], Safaeifar and Farshidianfar [44] models are shown in Fig. 10.

Although Flores et al. and Hu and Guo's models have similar behavior as the numerical model, they are not completely consistent with the numerical model for the low values of the coefficient of restitution.

Analyzing Fig. 10, it is clear that the optimal model proposed in this study and Safaeifar and Farshidianfar model are completely consistent with the numerical model in the whole range of the coefficient of restitution. So, the optimal model can be selected as the best contact force model for the collision between the two solid spheres in the whole range of the coefficient of restitution. Thus, this optimal model can be used in hard and soft impact problems.

It is important to point out that the optimal model is valid for the direct central and frictionless impacts. In perfectly elastic contacts when the coefficient of restitution is equal to one, the hysteresis damping factor is zero and the optimal model is equivalent to the Hertz model. When the coefficient of restitution is equal to zero, the hysteresis damping factor becomes infinite, which is logical from the physical view of the contact process.

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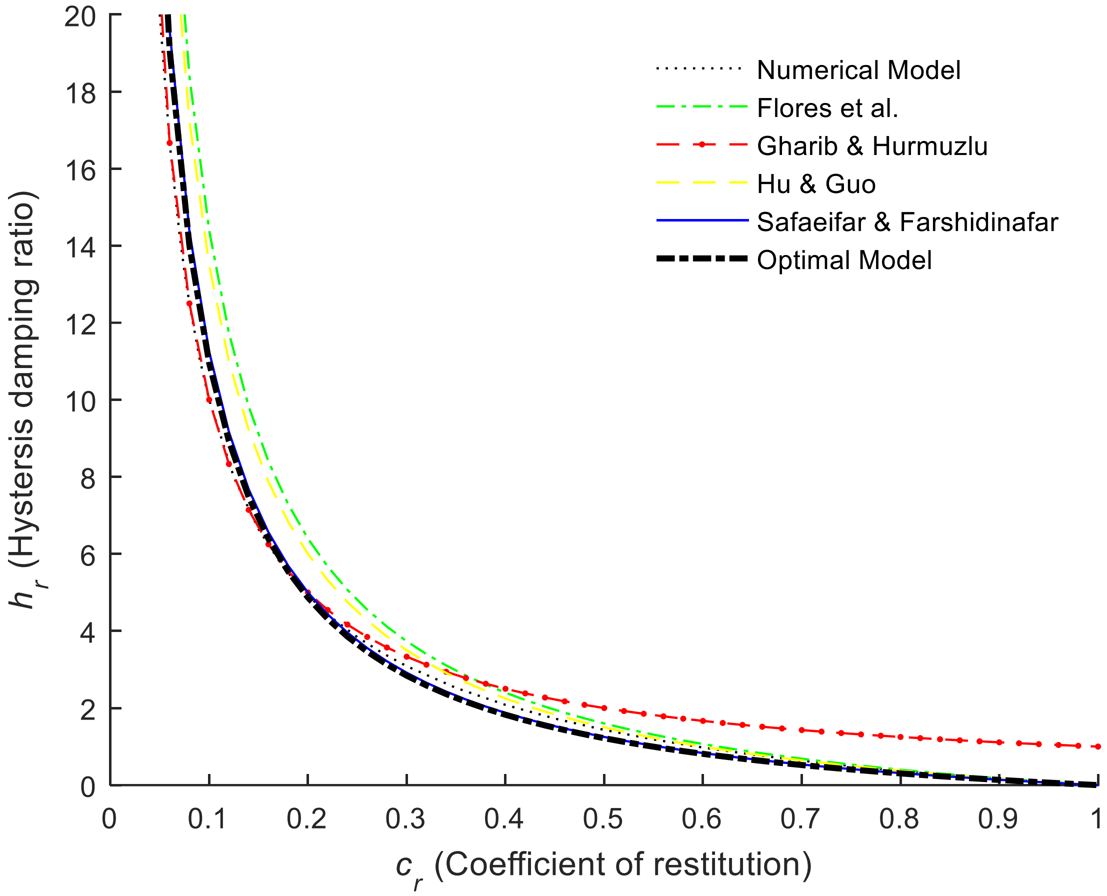

Figure 10. The hysteresis damping ratio versus the coefficient of restitution in the contact force models

5. CONCLUSION

In this paper, the optimal model of the contact of the collision between the two solid bodies has been derived using a hybrid meta heuristic optimization method. The optimal model can be applied directly for the impact analysis of the multibody dynamics. It has been stated by using the energy balance through the contact process. Using the classical kinetic energy principle, the variation in the kinetic energy is determined and, then a relation between the deformation and its velocity is obtained. For discovering the optimal model, the root mean square of the hysteresis damping ratio is minimized by ICACO. ICACO is a hybrid and powerful metaheuristic optimization method and due to the complexity of the problem and calculations, it is applied. Furthermore, the convergence rate is improved using ICACO, and is converged best results. Overall, the optimal model is perfectly proper for investigating the contact analysis for different coefficients of restitution. To sum up, this optimal model is efficient for soft and hard contact problems and also is an independent formula, and it can be applied directly for impact analysis of the multibody dynamic systems.

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